Solutions to Take-Home Quiz 2 (*September 17, 2010*)

Let L, T and C be matrices given below.

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 6 & 2 & 2 \\ 0 & 3 & 4 \end{bmatrix}, \qquad T = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 6 & 2 & -2 & 0 & 1 & 0 \\ 9 & -3 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

1. Compute the product LT. (Show work!)

   Sol.

$$LT = \begin{bmatrix} 0 & 1 & 0 \\ 6 & 2 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 0+6+0 & 0+2+0 & 0-2+0 \\ 6+12+18 & 6+4-6 & 6-4+2 \\ 0+18+36 & 0+6-12 & 0-6+4 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ 36 & 4 & 4 \\ 54 & -6 & -2 \end{bmatrix}$$

2. Find the reduced row echelon form of C. (Show work! Write operations as well in $[i;c]$, $[i,j]$, $[i,j;c]$ form.)

   Sol.

$$\begin{aligned} C \quad = \quad & \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 6 & 2 & -2 & 0 & 1 & 0 \\ 9 & -3 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[2,1;-6]} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -4 & -8 & -6 & 1 & 0 \\ 9 & -3 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[3,1;-9]} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -4 & -8 & -6 & 1 & 0 \\ 0 & -12 & -8 & -9 & 0 & 1 \end{bmatrix} \\ \xrightarrow{[3,2;-3]} & \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -4 & -8 & -6 & 1 & 0 \\ 0 & 0 & 16 & 9 & -3 & 1 \end{bmatrix} \xrightarrow{[2;-1/4]} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3/2 & -1/4 & 0 \\ 0 & 0 & 16 & 9 & -3 & 1 \end{bmatrix} \\ \xrightarrow{[1,2;-1]} & \begin{bmatrix} 1 & 0 & -1 & -1/2 & 1/4 & 0 \\ 0 & 1 & 2 & 3/2 & -1/4 & 0 \\ 0 & 0 & 16 & 9 & -3 & 1 \end{bmatrix} \xrightarrow{[3;-1/16]} \begin{bmatrix} 1 & 0 & -1 & -1/2 & 1/4 & 0 \\ 0 & 1 & 2 & 3/2 & -1/4 & 0 \\ 0 & 0 & 1 & 9/16 & -3/16 & 1/16 \end{bmatrix} \\ \xrightarrow{[1,4;-1]} & \begin{bmatrix} 1 & 0 & 0 & 1/16 & 1/16 & 1/16 \\ 0 & 1 & 2 & 3/2 & -1/4 & 0 \\ 0 & 0 & 1 & 9/16 & -3/16 & 1/16 \end{bmatrix} \xrightarrow{[2,4;-2]} \begin{bmatrix} 1 & 0 & 0 & 1/16 & 1/16 & 1/16 \\ 0 & 1 & 0 & 3/8 & 1/8 & -1/8 \\ 0 & 0 & 1 & 9/16 & -3/16 & 1/16 \end{bmatrix} \end{aligned}$$

3. Compute $T^{-1}LT$. (Show work!)

   Sol.

$$T^{-1}LT = \begin{bmatrix} 1/16 & 1/16 & 1/16 \\ 3/8 & 1/8 & -1/8 \\ 9/16 & -3/16 & 1/16 \end{bmatrix} \begin{bmatrix} 6 & 2 & -2 \\ 36 & 4 & 4 \\ 54 & -6 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

   Let D be the last matrix above, i.e., the diagonal matrix of size 3 with 6, 2, -2 on the diagonal. Since the columns of LT are scalar multiples of the columns of T by $6, 2, -2$ respectively, we have

$$LT = TD, \text{ or } T^{-1}LT = D.$$

   So this gives an alternate solution of this problem.