Take-Home **Quiz 7** (*Due at 7:00 p.m. on Fri. October 26, 2007*)

Division: **ID#:** **Name:**

Let A be a 5×5 matrix and B a 4×4 matrix given below.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 & x_5 \\ {x_1}^2 & {x_2}^2 & {x_3}^2 & {x_4}^2 & {x_5}^2 \\ {x_1}^3 & {x_2}^3 & {x_3}^3 & {x_4}^3 & {x_5}^3 \\ {x_1}^4 & {x_2}^4 & {x_3}^4 & {x_4}^4 & {x_5}^4 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ {x_1}^2 & {x_2}^2 & {x_3}^2 & {x_4}^2 \\ {x_1}^3 & {x_2}^3 & {x_3}^3 & {x_4}^3 \end{bmatrix}$$

1. Show that $\det(A) = (x_5 - x_1)(x_5 - x_2)(x_5 - x_3)(x_5 - x_4)\det(B)$. (Use the back of this sheet.)

2. Find $\det(A)$. (Solution only.)

3. Let $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ be a polynomial satisfying $f(-3) = 2$, $f(-1) = 5$, $f(2) = -3$, $f(3) = 0$ and $f(7) = 100$. Write down a system of linear equations to find a_0, a_1, a_2, a_3, a_4 and explain why the numbers a_0, a_1, a_2, a_3, a_4 are uniquely determined. Do not solve the equation!

4. Explain why there are infinitely many polynomials $g(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ satisfying $g(-3) = 2$, $g(-1) = 5$, $g(2) = -3$ and $g(3) = 0$.

Message 欄：あなたにとって一番たいせつな（または、たいせつにしたい）もの、ことはなんですか。そのほか、何でもどうぞ。[HP 掲載不可は明記のこと]