Solutions to Take-Home Quiz 1 (*September 14, 2007*)

$$\begin{cases} x_1 + x_3 - x_4 + 4x_5 = -3 \\ 2x_1 + 2x_3 - x_4 + 6x_5 = 1 \\ x_1 + x_3 + 2x_5 - x_6 = 5 \\ -x_1 - 2x_2 - 7x_3 - 4x_5 + x_6 = -7 \end{cases} \qquad B = \begin{bmatrix} 1 & 0 & 1 & -1 & 4 & 0 & -3 \\ 2 & 0 & 2 & -1 & 6 & 0 & 1 \\ 0 & -2 & -6 & 0 & -2 & 0 & -2 \\ -1 & -2 & -7 & 0 & -4 & 1 & -7 \end{bmatrix}$$

1. Find the augmented matrix A of the system of linear equations above.

   Sol.
   $$A = \begin{bmatrix} 1 & 0 & 1 & -1 & 4 & 0 & -3 \\ 2 & 0 & 2 & -1 & 6 & 0 & 1 \\ 1 & 0 & 1 & 0 & 2 & -1 & 5 \\ -1 & -2 & -7 & 0 & -4 & 1 & -7 \end{bmatrix}$$

2. The matrix B is obtained by applying an elementary row operation once to the augmented matrix A. Write the elementary row operation using the notation $[i; c]$, $[i, j]$, or $[i, j; c]$.

   Sol. $[3, 4; 1]$.

3. Find the reduced row echelon form of the augmented matrix A. (Solution only.)

   Sol. Apply the following consecutively in this order:
   $$[2, 1; -2],\ [4, 1; 1],\ [2, 3],\ [2, -\frac{1}{2}],\ [4, 2; 2],\ [4, 3; 1],\ [1, 3; 1].$$
   Then we have
   $$\begin{bmatrix} 1 & 0 & 1 & 0 & 2 & 0 & 4 \\ 0 & 1 & 3 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

   - There are many ways to obtain the reduced echelon form but the final matrix should be the same. When can we change the order of operations and when cannot?
   - Starting from the reduced row echelon form above, is it possible to obtain the matrix A back again by applying elementary row operations? Can you find the sequence of such elementary row operations from the one we obtained the reduced echelon form from A with a slight modification?

4. Find the solution of the system of linear equations. Use parameters if necessary.

   Sol.
   $$\begin{cases} x_1 = 4 - s - 2t \\ x_2 = 1 - 3s - t \\ x_3 = s \\ x_4 = 7 + 2t \\ x_5 = t \\ x_6 = -1 \end{cases}, \text{ or } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 7 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} -1 \\ -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ -1 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$
   s and t are parameters.